

Supergravity solutions for harmonic, static and flux S-branes

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ABSTRACT: We seek S-brane solutions in $D = 11$ supergravity which can be characterized by a harmonic function H on the flat transverse space. It turns out that the Einstein's equations force H to be a linear function of the transverse coordinates. The codimension one $H = 0$ hyperplane can be spacelike, timelike or null and the spacelike case reduces to the previously obtained SM2 or SM5 brane solutions. We then consider static S-brane configurations having smeared timelike directions where the transverse Lorentzian symmetry group is broken down to its maximal orthogonal subgroup. Assuming that the metric functions depend on a radial spatial coordinate, we construct explicit solutions in $D = 11$ supergravity which are non-supersymmetric and asymptotically flat. Finally, we obtain spacelike fluxbrane backgrounds which have timelike electric or magnetic fluxlines extending from past to future infinity.

KEYWORDS: Supergravity Models, p-branes.

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1. Introduction

Spacelike Sp -branes in string theory are topological defects which exist only for a moment in time. In perturbation theory they arise when the time coordinate obeys a Dirichlet boundary condition and in world sheet conformal field theory (CFT) they can be described by boundary states implementing the boundary conditions [1]. S-branes can also be considered as time dependent tachyonic kink solutions of unstable D-brane world-volume theories and they are expected to play the role of D-branes in realizing dS/CFT duality [2].

If S-branes really exist as genuine objects then one expects to have a corresponding supergravity description for them. Indeed, in [1, 3, 4] various time dependent supergravity solutions were constructed for S-branes in different dimensions. It is remarkable that the standard intersection rules for usual p -branes also arise for S-branes [5] and one can further find solutions for nonstandard intersections [6] or intersections of p -branes with S-branes [7]. By analytical continuations of black holes, completely regular S-brane solutions can also be obtained [8–10] (for other works on S-brane solutions see, e.g., [11–20]).

Sp -branes in D -dimensions have transverse $SO(1, D-p-2)$ R-symmetry. The Lorentz invariant combination of the transverse coordinates gives a null direction, and as shown in [4], one can construct solutions corresponding to interior and exterior regions of the light cone which can be foliated by hyperbolic and de Sitter spaces, respectively. On the other hand, for some physical applications it would be interesting to consider solutions where the Lorentzian symmetry is broken (in terms of Euclidean gauge theory living on the S-brane that would correspond to breaking of R-symmetry by giving vacuum expectation values to scalars). One aim of this paper is to study this possibility.

It is well known that usual p -brane solutions are characterized by harmonic functions. In general the corresponding antisymmetric tensor F obeys $dF = 0$ and $d * F = 0$, and these can be satisfied naturally using harmonic functions by writing $F \sim dH$ or $*F \sim dH$

for electric or magnetic solutions, respectively. As far as the form fields are concerned S-branes are not very different than p -branes. On the other hand, harmonic superposition of p -branes is possible due to supersymmetry, which is absent for S-branes. In the next section we seek S-brane solutions which are characterized by harmonic functions. If harmonic S-branes exist then there should be a way of avoiding a superposition principle. Indeed, as we will see below, the form equations can be satisfied naturally by harmonic functions but the Einstein's equations demand linearity on the transverse space so that superposition of different functions does not yield a new solution but simply modifies the parameters in the total function. In the same section we also generalize harmonic S-branes where the flat transverse spaces are replaced by general Ricci flat manifolds.

For usual p -brane backgrounds, one common procedure is to smear a transverse coordinate by putting an array of parallel branes in that direction. In principle supersymmetry is required for smearing and it is not clear how this may be achieved for S-branes. In any case, one can still imagine a solution for S-branes distributed uniformly in time (one can argue that stability is not a question here since "time evolution" is fixed by hand from the beginning like imposing a boundary condition). For such a background there is a time translation symmetry and the corresponding solution should be static. In section 3, we construct explicit solutions in $D = 11$ supergravity which can be thought to represent time smeared S-branes where the transverse Lorentzian R-symmetry group is broken down to its maximal orthogonal subgroup. As we will see below these backgrounds are non-supersymmetric, asymptotically flat, generically singular in the interior but support finite ADM masses per unit Euclidean S-brane volumes.

Finally in section 4 we obtain spacelike fluxbrane solutions in $D = 11$ supergravity theory. Generically, a fluxbrane background has antisymmetric tensor field components tangent to the transverse coordinates. Although the fluxlines have infinite extent the total charge is finite. This is interpreted as the confinement of the fluxlines by their own gravitational field. The classical example is the Melvin solution of 4-dimensional Einstein-Maxwell gravity [21] which describes a flux 1-brane. Later various higher dimensional generalizations of fluxbranes were constructed in the literature (see e.g. [22–29]). As the spacelike branes have recently attracted some attention in string theory, one may wonder if there are solutions for spacelike fluxbranes. In such a background the transverse space is Lorentzian and therefore the antisymmetric tensor field should have a component along time direction. As we will see one can construct time dependent solutions with this property, representing spacelike fluxbranes. Similar to usual timelike fluxbranes, the spacelike solutions have fluxlines extending from past to future timelike infinity but the total flux still converges despite the infinite range. This shows that the confinement of fluxlines by gravity also works for spacelike solutions.

2. Harmonic S-branes

In this section our aim is to construct S-brane solutions characterized by harmonic functions. As an example let us consider SM2 brane in eleven dimensions. The equations of

motion for the bosonic fields of $D = 11$ supergravity can be written as

$$R_{MN} = \frac{1}{3} F_{MPQR} F_N{}^{PQR} - \frac{1}{36} g_{MN} F_{PQRS} F^{PQRS}$$

$$dF = 0, \quad d * F = F \wedge F. \tag{2.1}$$

We consider the following metric¹

$$ds^2 = e^{2A} \delta_{ab} dx^a dx^b + e^{2B} \eta_{\mu\nu} dy^\mu dy^\nu, \tag{2.2}$$

where $a, b = 1, 2, 3$ and $\mu, \nu = 0, \dots, 7$. The coordinate dependencies of the metric functions A and B are assumed to be of the form $A = A(H)$ and $B = B(H)$, where $H = H(y)$ is a function on the transverse space. For the antisymmetric tensor we take

$$F_{abc\mu} = \frac{1}{2} e^{-7B} \epsilon_{abc} \partial_\mu H. \tag{2.3}$$

Note that $dF = 0$ is identically satisfied and the antisymmetric tensor equation $d * F = 0$ gives

$$\partial_\mu \partial^\mu H = 0. \tag{2.4}$$

Using (2.4) in Einstein's equations we find

$$\ddot{A} + 3\dot{A}^2 + 6\dot{A}\dot{B} + \frac{e^{-12B}}{3} = 0, \tag{2.5}$$

$$\ddot{B} + 6\dot{B}^2 + 3\dot{A}\dot{B} - \frac{e^{-12B}}{6} = 0, \tag{2.6}$$

$$\ddot{A} + 2\ddot{B} + \dot{A}^2 - 2\dot{B}^2 - 2\dot{A}\dot{B} + \frac{e^{-12B}}{6} = 0, \tag{2.7}$$

$$\dot{A} + 2\dot{B} = 0, \tag{2.8}$$

where dot denotes differentiation with respect to the argument (i.e. H). Eq. (2.5) follows from the worldvolume directions $(M, N) = (a, b)$ in (2.1) and the terms coming from $(M, N) = (\mu, \nu)$ components fall into three different groups with coefficients $\eta_{\mu\nu}(\partial H)^2$, $\partial_\mu H \partial_\nu H$ and $\partial_\mu \partial_\nu H$. Setting them to zero separately² gives (2.6), (2.7) and (2.8), respectively. Eq. (2.8) yields $A = -2B$ and then (2.7) implies

$$\dot{B}^2 + \frac{e^{-12B}}{36} = 0, \tag{2.9}$$

which shows that the solution space is *empty*. This result is not surprising since otherwise harmonic superposition of S-branes would be possible, which is odd in the absence of supersymmetry.

¹As shown in [30] as far as the supergravity solutions are concerned the "internal" flat spaces, spheres or hyperboloids can be replaced by arbitrary Ricci flat, positively or negatively curved Einstein spaces, respectively. Therefore in (2.2), for instance, the flat space spanned by (x^1, x^2, x^3) can be replaced by any Ricci flat manifold.

²At this point we do not want to impose any additional condition on H , thus the functions $\eta_{\mu\nu}(\partial H)^2$, $\partial_\mu H \partial_\nu H$ and $\partial_\mu \partial_\nu H$ are assumed to be linearly independent.

One way of proceeding is to consider type IIA* or IIB* string theories studied in [31, 32] which can be obtained by timelike T-dualities from type IIA and IIB strings, respectively. In supergravity actions a timelike T-duality alters the signs of the kinetic terms of the antisymmetric tensor fields. Lifting IIA* theory to eleven dimensions and considering the same ansatz (2.2) and (2.3) in this framework, (2.9) becomes

$$\dot{B}^2 - \frac{e^{-12B}}{36} = 0, \tag{2.10}$$

which implies $B = (\ln H)/6$. Using also $A = -2B$, one ends up with the Wick rotated Euclidean p -branes studied in [31].

Another possibility is to impose $\partial_\mu \partial_\nu H = 0$ so that (2.8) is dropped from the equation system. In this case the harmonic function H becomes

$$H = c_\mu y^\mu + c, \tag{2.11}$$

where (c_μ, c) are constants. A systematic way of solving equations like (2.5)-(2.7) was discussed in [33]. Adding (2.5) and two times (2.6) yields

$$(\ddot{A} + 2\ddot{B}) + 3(\dot{A} + 2\dot{B})^2 = 0, \tag{2.12}$$

which can be solved as $A + 2B = (\ln H)/3$. Using this in (2.6) one obtains

$$\ddot{B} + \frac{\dot{B}}{H} - \frac{e^{-12B}}{6} = 0, \tag{2.13}$$

which has the solution

$$B = \frac{1}{12} \ln \left[b H^2 \cosh^2 \left[\frac{\ln(\pm H)}{\sqrt{b}} \right] \right], \tag{2.14}$$

where b is a constant. Eq. (2.7), on the other hand, acts like a constraint equation which fixes $b = 3/7$. The end result is that

$$\begin{aligned} A &= -\frac{1}{6} \ln \left[\frac{3}{7} \cosh^2 \left[\sqrt{\frac{7}{3}} \ln(\pm H) \right] \right], \\ B &= \frac{1}{12} \ln \left[\frac{3H^2}{7} \cosh^2 \left[\sqrt{\frac{7}{3}} \ln(\pm H) \right] \right], \end{aligned} \tag{2.15}$$

where \pm signs are for $H > 0$ and $H < 0$ regions respectively.

Let Σ_H be the codimension one $H = 0$ hyperplane which can be spacelike, timelike or null. In each case one can apply suitable Lorentz transformations and translations to set $H = c_0 t$, $H = c_1 y^1$ or $H = c_\pm (t \pm y^1)$, respectively. After a coordinate transformation $\tilde{t} = \ln(c_0 t)$, it is not difficult to see that the spacelike case is identical to the time dependent SM2 brane solution with a flat transverse space. Here one discovers two additional backgrounds corresponding to timelike and null planes. Note that superposition of different harmonic functions (i.e. when $H = \sum_i H_i$) does not give a new solution but simply modifies the constants (c_μ, c) . This is somehow expected in the absence of supersymmetry.

For the null plane, (2.6) is redundant since it is actually multiplied by $\eta_{\mu\nu}(\partial H)^2 = 0$ and the solution space is larger. However, superposition with the spacelike and the timelike backgrounds is not possible for this general class. So we impose (2.6) as an additional equation to ensure superposition.

The electric charge of SM2 brane is given by

$$Q = \int *F = \int_{\Sigma_H} \hat{*} dH, \tag{2.16}$$

where $\hat{*}$ is the 8-dimensional Hodge dual on the flat transverse space. The integrand is a constant and the total charge diverges. The fluxlines are confined on Σ_H , but they have constant magnitude and infinite extent (of course it is possible to compactify Σ_H to get finite charge). The metric is singular on Σ_H and at infinity along the perpendicular direction to Σ_H , i.e. when $H \rightarrow \pm\infty$.

It is easy to repeat the above construction for the magnetic SM5 brane which has the following metric

$$ds^2 = e^{2A(H)} \delta_{ab} dx^a dx^b + e^{2B(H)} \eta_{\mu\nu} dy^\mu dy^\nu, \tag{2.17}$$

where $a, b = 1, \dots, 6$ and $\mu, \nu = 0, \dots, 4$. The antisymmetric tensor can be taken as

$$F = \frac{1}{2} \hat{*} dH, \tag{2.18}$$

where $\hat{*}$ is the Hodge dual on the flat 5-dimensional transverse space. The form equation $d * F = 0$ is identically satisfied and $dF = 0$ implies $\partial_\mu \partial^\mu H = 0$. On the other hand the Einstein's equations require $\partial_\mu \partial_\nu H = 0$ and thus H is linear as in (2.11). The metric functions can be solved as

$$\begin{aligned} A &= -\frac{1}{12} \ln \left[\frac{3}{8} \cosh^2 \left[\sqrt{\frac{8}{3}} \ln(\pm H) \right] \right], \\ B &= \frac{1}{6} \ln \left[\frac{3 H^2}{8} \cosh^2 \left[\sqrt{\frac{8}{3}} \ln(\pm H) \right] \right]. \end{aligned} \tag{2.19}$$

Again \pm signs are for $H > 0$ and $H < 0$ regions respectively. The solution carries a magnetic charge

$$Q = \int F = \int_{\Sigma_H} \hat{*} dH, \tag{2.20}$$

which diverges since the constant magnetic fluxlines extent on Σ_H to infinity. When Σ_H is spacelike one can apply a Lorentz transformation to set $H = c_0 t$ and the solution becomes the usual time dependent SM5 brane with a flat transverse space after the coordinate change $\tilde{t} = \ln(c_0 t)$.

For both SM2 and SM5 branes R-symmetry groups are determined by the isometries of the corresponding plane Σ_H . For instance for SM2 brane the symmetry groups are $ISO(7)$, $ISO(1, 6)$ and $\mathcal{R} \times ISO(6)$ when Σ_H is spacelike, timelike and null, respectively.

It is known that time dependent solutions (i.e. when Σ_H is spacelike in our case) cannot be supersymmetric since the "Hamiltonian", which can be written as the anti-commutator of supercharges, is not a "constant". To see if there is any unbroken supersymmetry for

other cases let us consider the Killing spinor equation

$$D_M \epsilon \equiv \left[\nabla_M + \frac{1}{144} (\Gamma^{PQRS}{}_M - 8 \delta_M^P \Gamma^{QRS}) F_{PQRS} \right] \epsilon = 0. \quad (2.21)$$

We check out the integrability condition

$$D_{[M} D_{N]} \epsilon = 0 \quad (2.22)$$

and find that there is no Killing spinor when Σ_H is timelike or spacelike. For instance in the SM2 brane solution $D_{[a} D_{b]} \epsilon = 0$ implies

$$\left[\dot{A}^2 - \frac{e^{-12B}}{9} \right] (\partial^\mu H \partial_\mu H) \Gamma_{ab} \epsilon = 0. \quad (2.23)$$

Using (2.15) one finds that $(\dot{A}^2 - e^{-12B}/9) \neq 0$ and thus $\epsilon = 0$ since $\partial^\mu H \partial_\mu H \neq 0$.

For the null solution (2.23) is identically satisfied and the integrability condition $D_{[a} D_{\nu]} \epsilon = 0$ implies

$$\Gamma^\mu \partial_\mu H \epsilon = 0. \quad (2.24)$$

Imposing (2.24), one can find Killing spinors of the form

$$\epsilon = \begin{cases} [\Gamma^\mu \partial_\mu H] e^{f+g\Gamma^{123}} \epsilon_0 & \text{SM2,} \\ [\Gamma^\mu \partial_\mu H] e^{f+g\Gamma^{123456}} \epsilon_0 & \text{SM5,} \end{cases} \quad (2.25)$$

where ϵ_0 is a constant spinor, $f = B/2$, $\dot{g} = e^{-6B}/12$ for SM2 and $\dot{g} = e^{-3B}/12$ for SM5 branes. Therefore the null backgrounds preserve 16 supersymmetries of $D = 11$ supergravity.

We should remark that the above solutions should be considered on $H > 0$ or $H < 0$ regions separately. If one tries to use them in the whole space then extra delta function singularities appear, which arise since the metric functions (2.15) and (2.19) are discontinuous on Σ_H . For both SM2 and SM5 branes the Ricci tensor components containing second derivatives of the functions A and B can be found as

$$\begin{aligned} R_{ab} &= -e^{-2B} \ddot{A} (\partial_\lambda H) (\partial^\lambda H) \delta_{ab} + \dots \\ R_{\mu\nu} &= -e^{-2B} \ddot{B} (\partial_\lambda H) (\partial^\lambda H) \eta_{\mu\nu} + \dots \end{aligned} \quad (2.26)$$

where the dotted terms involve only the first derivatives plus the second derivatives of $(A + 2B)$ for SM2 and $(2A + B)$ for SM5 branes which are however continuous across Σ_H . From (2.15) and (2.19) one finds as $H \rightarrow 0$ that

$$e^{-2B} \ddot{A} \sim e^{-2B} \ddot{B} \sim \begin{cases} H^{(-4+\sqrt{7/3})/3} \delta(H) + \dots & \text{SM2,} \\ H^{(-5+2\sqrt{8/3})/3} \delta(H) + \dots & \text{SM5.} \end{cases} \quad (2.27)$$

Therefore to obtain supergravity solutions in the whole region, these backgrounds should be supplemented by additional delta function sources on Σ_H which may arise from the coupling of elementary S-branes to the supergravity fields. In this case, however, one would

expect the antisymmetric tensor fields also to be modified. To achieve this one may replace (2.11) with $H = |c_\mu y^\mu + c|$, which is the Greens function in one dimension perpendicular to Σ_H . Although there is no need for the negative signs in the metric functions (2.15) and (2.19), the second derivatives of H now yield additional delta functions in the Ricci tensor. Similarly, unless c_μ is a null vector the form equations are modified such that $d*F \sim \delta(H)$ for SM2 and $dF \sim \delta(H)$ for SM5 branes.

As a final comment let us point out that it is possible to replace the flat transverse space with a curved one

$$\eta_{\mu\nu} dy^\mu dy^\nu \rightarrow d\mathcal{L}^2, \tag{2.28}$$

where \mathcal{L} is an arbitrary Ricci flat Lorentzian space. It is not difficult to verify that the SM2 and SM5 brane backgrounds (2.15) and (2.19) satisfy field equations provided H obeys

$$\hat{\nabla}_\mu \hat{\nabla}_\nu H = 0, \tag{2.29}$$

where $\hat{\nabla}_\mu$ is the covariant derivative on \mathcal{L} . If such a function H exists on \mathcal{L} then it yields a covariantly constant vector field $k_\mu = \hat{\nabla}_\mu H$. Conversely, for any covariantly constant vector field k_μ one can find a *local* function H such that $k_\mu = \hat{\nabla}_\mu H$. Moreover when k_μ is hypersurface orthogonal then H is globally well defined. Therefore, one can write down a solution for each covariantly constant vector field on \mathcal{L} where the metric functions (2.15) or (2.19) depend on the (local) potential H and the antisymmetric tensor fields become $*F = \hat{*}k$ for SM2 and $F = \hat{*}k$ for SM5 branes, where $\hat{*}$ is Hodge dual on \mathcal{L} and k is the one-form corresponding to k_μ . The electric or the magnetic charge of this solution can be calculated as

$$Q = \int \hat{*}k \tag{2.30}$$

which gives a finite result when the cycle dual to one-form k is compact.

3. Static S-branes

It is well known that a transverse direction to a p -brane worldvolume can be smeared out by placing a continuum array of parallel branes in that direction. This can be achieved due to supersymmetry which ensures stability. It is not clear whether one can place two parallel S-branes separated by a finite time interval and thus whether smearing is possible. Assuming this can be done one can consider an infinite array of S-branes. It can be claimed that the stability of this system is not an issue since the time evolution is dictated by hand like imposing a boundary condition. In this section we construct explicit solutions in $D = 11$ supergravity which can be thought to represent smeared S-branes.

Let us consider SM2 brane first. It is clear that the supergravity solution should be static. Moreover the transverse $SO(1, 7)$ symmetry should be broken down to $SO(7)$ subgroup and one can introduce a radial transverse coordinate. Thus the metric can be taken as

$$ds^2 = e^{2A} (dx_1^2 + dx_2^2 + dx_3^2) - e^{2B} dt^2 + e^{2C} dr^2 + e^{2D} d\Omega_6^2, \tag{3.1}$$

where Ω_6 is the unit 6-sphere and the unknown functions depend on r . The three-form potential \mathcal{A} should couple to the Euclidean world volume and can be written as $\mathcal{A}_{abc} =$

$f(r)\epsilon_{abc}$. Solving the antisymmetric tensor equation $d * F = 0$ one finds (the indices refer to the tangent space)

$$F_{abcr} = \frac{k}{2} e^{-B-6D} \epsilon_{abc}, \quad (3.2)$$

where k is an integration constant. Let us point out that despite the similarity the above background differs from brane anti-brane systems studied in the literature (see e.g. in [33–35]). The main distinction is in the choice of the antisymmetric tensor which is constructed here to represent a Euclidean brane.

Imposing the gauge

$$C = 3A + B + 6D, \quad (3.3)$$

one finds the following second order equations

$$\begin{aligned} A'' &= -\frac{k^2}{3} e^{6A}, \\ B'' &= \frac{k^2}{6} e^{6A}, \\ D'' &= 5 e^{6A+2B+10D} + \frac{k^2}{6} e^{6A}, \end{aligned} \quad (3.4)$$

together with a first order constraint

$$C'^2 - 3A'^2 - B'^2 - 6D'^2 - \frac{k^2}{2} e^{6A} - 30 e^{6A+2B+10D} = 0, \quad (3.5)$$

where prime denotes differentiation with respect to r . The system (3.4) can be integrated step by step starting from A to yield

$$\begin{aligned} A &= -\frac{1}{3} \ln [k \cosh r], \\ B &= \frac{1}{6} \ln [k \cosh r] + c_1 r, \\ D &= \frac{1}{6} \ln [k \cosh r] - \frac{1}{5} \ln \left[\frac{10}{c_2} \sinh \left(\frac{c_2 r}{2} \right) \right] - \frac{c_1 r}{5}, \end{aligned} \quad (3.6)$$

where c_1 and c_2 are integration constants (we scale r to eliminate one constant in A). The constraint (3.5) imposes

$$c_2^2 = \frac{5}{3} + 4c_1^2. \quad (3.7)$$

One should choose $c_2 > 0$ to have a well defined metric function D in (3.6) and c_1 can be positive, negative or zero. Introducing a new radial coordinate

$$\tilde{r} = \left[\tanh \left(\frac{c_2 r}{4} \right) \right]^{-1/5}, \quad (3.8)$$

the metric becomes

$$ds^2 = e^{2A} (dx_1^2 + dx_2^2 + dx_3^2) - e^{2B} dt^2 + \frac{e^{2D}}{\tilde{r}^2} [d\tilde{r}^2 + \tilde{r}^2 d\Omega_6^2], \quad (3.9)$$

where the functions A , B and D are still given by (3.6) with $r = 4 \operatorname{arctanh}(\tilde{r}^{-5})/c_2$. In (3.9) one can introduce Cartesian coordinates in the flat space parametrized by (\tilde{r}, Ω_6) and $S^0(7)$ symmetry, which acts as rotations around the fixed origin, becomes manifest.

The above solution is asymptotically flat as $\tilde{r} \rightarrow \infty$ (or $r \rightarrow 0$) where the metric functions can be expanded as

$$\begin{aligned} e^A &= k^{-1/3} \left[1 + \mathcal{O}\left(\frac{1}{\tilde{r}^{10}}\right) \right] \\ e^B &= k^{1/6} \left[1 + \frac{4c_1}{c_2} \frac{1}{\tilde{r}^5} + \mathcal{O}\left(\frac{1}{\tilde{r}^{10}}\right) \right] \\ e^D &= k^{1/6} \left(\frac{c_2}{20}\right)^{1/5} \tilde{r} \left[1 - \frac{4c_1}{5c_2} \frac{1}{\tilde{r}^5} + \mathcal{O}\left(\frac{1}{\tilde{r}^{10}}\right) \right]. \end{aligned} \tag{3.10}$$

Here $1/\tilde{r}^5$ fall off is expected since the spatial transverse space is 7-dimensional. The solution supports finite ADM mass (per unit Euclidean volume) which is given by

$$M = \Omega_6 k^{5/6} \frac{3c_1}{5\kappa^2}, \tag{3.11}$$

where κ is the gravitational coupling constant and Ω_6 is the volume of unit 6-sphere. To get positive mass one should choose $c_1 > 0$. Although c_2 does not contribute to ADM mass it cannot be scaled away.

To analyze the interior region near $\tilde{r} \rightarrow 1$ (or $r \rightarrow \infty$) let us introduce a new coordinate u with

$$u = \frac{e^{-cr}}{c}, \quad c = \frac{3c_2 + c_1}{5} - \frac{1}{6}. \tag{3.12}$$

We note that due to (3.7) c is always a positive constant. In the limit $\tilde{r} \rightarrow 1$, (or $r \rightarrow \infty$), $u \rightarrow 0$ and the metric becomes

$$ds^2 \rightarrow u^{2/(3c)} (dx_1^2 + dx_2^2 + dx_3^2) - u^{-(1+6c_1)/(3c)} dt^2 + du^2 + u^{2-c_2/c} d\Omega_6^2, \tag{3.13}$$

which is singular at $u = 0$ since for instance the coefficient of dx_1^2 vanishes. Although the solution is asymptotically flat and supports finite ADM mass it contains a naked singularity in the interior.

To see whether there is any unbroken supersymmetry in the system we check out the integrability condition (2.22). A simple calculation shows that $D_{[a}D_{b]}\epsilon = 0$ implies

$$\Gamma^{at} (f + g\Gamma^{123}) \epsilon = 0 \quad \Rightarrow \quad \epsilon = 0, \tag{3.14}$$

where $f = -A'B'e^{A+B-2C}/4$ and $g = kB'e^{A-C-6D}/24$. Since $(f + g\Gamma^{123})$ is an invertible matrix the solution does not preserve any supersymmetry.

It is possible to smear some of the transverse directions and consider a metric of the form³

$$ds^2 = e^{2A} (dx_1^2 + dx_2^2 + dx_3^2) - e^{2B} dt^2 + e^{2C} dr^2 + e^{2D_1} (dy_1^2 + \dots + dy_m^2) + e^{2D_2} d\Omega_n^2, \tag{3.15}$$

³One can consider the most general case where each transverse y -coordinate in (3.15) is multiplied by a different function. It turns out that the equation system can still be decoupled in this general background.

where $m + n = 6$ and $n \geq 2$. The antisymmetric tensor should be modified as

$$F_{abcr} = \frac{k}{2} e^{-B-mD_1-nD_2} \epsilon_{abc}. \quad (3.16)$$

Imposing the gauge

$$C = 3A + B + mD_1 + nD_2, \quad (3.17)$$

the differential equations are decoupled and the metric functions can be integrated step by step to yield

$$\begin{aligned} A &= -\frac{1}{3} \ln [k \cosh r], \\ B &= \frac{1}{6} \ln [k \cosh r] + c_1 r, \\ D_1 &= \frac{1}{6} \ln [k \cosh r] + c_2 r, \\ D_2 &= \frac{1}{6} \ln [k \cosh r] - \frac{1}{(n-1)} \ln \left[\frac{2(n-1)}{c_3} \sinh \left(\frac{c_3 r}{2} \right) \right] - \frac{c_1 + (6-n)c_2}{(n-1)} r, \end{aligned} \quad (3.18)$$

where the constants obey

$$n c_3^2 - 20 m c_2^2 - 4 n c_1^2 - 8 m c_1 c_2 + 2 - 2n = 0. \quad (3.19)$$

If one chooses $c_1 = c_2$ then $B = D_1$, i.e. the metric factors multiplying t and (y_1, \dots, y_m) coordinates become equal. In this case there is an extra $ISO(1, m)$ symmetry acting on the space spanned by (t, y_1, \dots, y_m) . The proper radial coordinate is given by $\tilde{r} = \tanh(c_3 r/4)^{-1/(n-1)}$ such that in (3.15)

$$e^{2C} dr^2 + e^{2D_2} d\Omega_n^2 \rightarrow \frac{e^{2D_2}}{\tilde{r}^2} (d\tilde{r}^2 + \tilde{r}^2 d\Omega_n^2). \quad (3.20)$$

The solution is still asymptotically flat as $\tilde{r} \rightarrow \infty$ (or $r \rightarrow 0$) and singular in the interior as $\tilde{r} \rightarrow 1$ (or $r \rightarrow \infty$). The metric functions fall off at least with the power $\tilde{r}^{-(n-1)}$ so that the ADM mass is finite.

Let us now consider the static SM5 brane solution. The metric and the antisymmetric tensor are given by

$$\begin{aligned} ds^2 &= e^{2A} (dx_1^2 + \dots + dx_6^2) - e^{2B} dt^2 + e^{2C} dr^2 + e^{2D} d\Omega_3^2, \\ F_{\alpha\beta\gamma t} &= \frac{k}{2} e^{-B-3D} \epsilon_{\alpha\beta\gamma}, \end{aligned} \quad (3.21)$$

where the indices α, β, γ refer to the tangent space on Ω_3 and the unknown functions depend on r . The field equations can be solved to get

$$\begin{aligned} A &= -\frac{1}{6} \ln [k \cosh r], \\ B &= \frac{1}{3} \ln [k \cosh r] + c_1 r, \\ C &= \frac{1}{3} \ln [k \cosh r] - \frac{3}{2} \ln \left[\frac{4}{c_2} \sinh \left(\frac{c_2 r}{2} \right) \right] - \frac{c_1 r}{2}, \\ D &= \frac{1}{3} \ln [k \cosh r] - \frac{1}{2} \ln \left[\frac{4}{c_2} \sinh \left(\frac{c_2 r}{2} \right) \right] - \frac{c_1 r}{2}, \end{aligned} \quad (3.22)$$

where the constants obey

$$c_2^2 = \frac{4}{3} + 4c_1^2. \quad (3.23)$$

Introducing the proper radial coordinate

$$\tilde{r} = \left[\tanh\left(\frac{c_2 r}{4}\right) \right]^{-1/2}, \quad (3.24)$$

the metric becomes

$$ds^2 = e^{2A} (dx_1^2 + \dots + dx_6^2) - e^{2B} dt^2 + \frac{e^{2D}}{\tilde{r}^2} (d\tilde{r}^2 + \tilde{r}^2 d\Omega_3^2). \quad (3.25)$$

The $SO(4)$ R-symmetry acts on the flat space spanned by (\tilde{r}, Ω_3) . The solution is asymptotically flat as $\tilde{r} \rightarrow \infty$ (or $r \rightarrow 0$) where the functions e^A , e^B and $\tilde{r}e^D$ fall off with the powers $1/\tilde{r}^4$, $1/\tilde{r}^2$ and $1/\tilde{r}^2$, respectively. The ADM mass (per unit Euclidean volume) can be calculated as

$$M = \Omega_3 k^{2/3} \frac{3c_1}{4\kappa^2}, \quad (3.26)$$

where κ is the gravitational coupling constant and Ω_3 is the volume of unit 3-sphere. The metric is singular as $\tilde{r} \rightarrow 1$ (or $r \rightarrow \infty$, $u \rightarrow 0$):

$$ds^2 \rightarrow u^{1/(3c)} (dx_1^2 + dx_2^2 + dx_3^2) - u^{-(2+6c_1)/(3c)} dt^2 + du^2 + u^{2-c_2/c} d\Omega_6^2, \quad (3.27)$$

where $u = e^{-cr}/c$ and $c = -1/3 + c_1/2 + 3c_2/4$ is a positive constant. We check out that there is no Killing spinor on this background and thus the solution is not supersymmetric.

Finally let us note the smeared solution where $\Omega_3 \rightarrow \mathcal{R} \times \Omega_2$. The fields are given by

$$\begin{aligned} ds^2 &= e^{2A} (dx_1^2 + \dots + dx_6^2) - e^{2B} dt^2 + e^{2C} dr^2 + e^{2D_1} dy^2 + e^{2D_2} d\Omega_2^2, \\ F_{\alpha\beta\gamma t} &= \frac{k}{2} e^{-B-D_1-2D_2} \epsilon_{\alpha\beta\gamma}, \end{aligned} \quad (3.28)$$

where

$$\begin{aligned} A &= -\frac{1}{6} \ln [k \cosh r], \\ B &= \frac{1}{3} \ln [k \cosh r] + c_1 r, \\ C &= \frac{1}{3} \ln [k \cosh r] - 2 \ln \left[\frac{2}{c_3} \sinh \left(\frac{c_3 r}{2} \right) \right] - (c_1 + c_2)r, \\ D_1 &= \frac{1}{3} \ln [k \cosh r] + c_2 r, \\ D_2 &= \frac{1}{3} \ln [k \cosh r] - \ln \left[\frac{2}{c_3} \sinh \left(\frac{c_3 r}{2} \right) \right] - (c_1 + c_2)r, \end{aligned} \quad (3.29)$$

and the constants c_1, c_2, c_3 obey

$$c_3^2 - 4c_2^2 - 4c_1^2 - 4c_1c_2 - 1 = 0. \quad (3.30)$$

The proper radial coordinate is $\tilde{r} = \tanh(c_3 r/4)^{-1}$. The metric is asymptotically flat as $\tilde{r} \rightarrow \infty$ and it is singular in the interior as $\tilde{r} \rightarrow 1$.

4. Spacelike fluxbranes

The main example of a fluxbrane is the Melvin background of 4-dimensional Einstein-Maxwell gravity [21] which is given by

$$\begin{aligned}
 ds^2 &= \left(1 + \frac{B^2 r^2}{4}\right)^2 [-dt^2 + dz^2 + dr^2] + \frac{r^2}{\left(1 + \frac{B^2 r^2}{4}\right)^2} d\phi^2 \\
 F &= \frac{B}{\left(1 + \frac{B^2 r^2}{4}\right)^2} r dr \wedge d\phi.
 \end{aligned}
 \tag{4.1}$$

The constant B is the magnetic field strength on the axis $r = 0$. The total flux can be calculated as

$$\int_{R^2} F = \frac{4\pi}{B},
 \tag{4.2}$$

which is finite although the lines have infinite extent. This is interpreted as the confinement of the magnetic fluxlines by gravity. As r increases the orbits of ϕ become small and the solution resembles a teardrop with an infinite tail.

To get a spacelike fluxbrane in this theory we perform the following analytical continuations

$$r \rightarrow i t, \quad t \rightarrow i y, \quad B \rightarrow i B, \quad \phi \rightarrow i \phi,
 \tag{4.3}$$

which give

$$\begin{aligned}
 ds^2 &= \left(1 + \frac{B^2 t^2}{4}\right)^2 [-dt^2 + dy^2 + dz^2] + \frac{t^2}{\left(1 + \frac{B^2 t^2}{4}\right)^2} d\phi^2 \\
 F &= \frac{B}{\left(1 + \frac{B^2 t^2}{4}\right)^2} t dt \wedge d\phi.
 \end{aligned}
 \tag{4.4}$$

This can be interpreted as a Euclidean flux 1-brane which has the worldvolume coordinates (y, z) . The fluxlines now extend in time to infinity but the integral of F is still finite. The geometry is locally flat as $t \rightarrow 0$ and the orbits of ϕ diminish as $t \rightarrow \infty$, resembling a teardrop extending in time.

Our aim in this section is to construct higher dimensional generalizations of spacelike fluxbranes. Let us consider flux SM3 brane first which has the metric

$$ds^2 = e^{2A} (dx_1^2 + \dots + dx_4^2) - e^{2B} dt^2 + e^{2C} d\Sigma_6^2,
 \tag{4.5}$$

where the functions A, B, C depend on t and Σ_6 is a Ricci flat (preferably compact) space. For the antisymmetric tensor we take

$$F_{abcd} = \frac{k}{2} e^{-4A} \epsilon_{abcd},
 \tag{4.6}$$

so that the form equations are identically satisfied. This is an electrically charged solution and the fluxlines (related to $*F$) both extend in time and wrap over Σ_6 .

Imposing the gauge $B = 4A + 6C$ the field equations become

$$\begin{aligned}\ddot{A} &= \frac{k^2}{3} e^{12C} \\ \ddot{C} &= -\frac{k^2}{6} e^{12C} \\ 10\dot{A}^2 + 30\dot{C}^2 + 48\dot{A}\dot{C} - \frac{k^2}{2} e^{12C} &= 0,\end{aligned}\tag{4.7}$$

where dot denotes time derivative. These equations can be integrated to get

$$\begin{aligned}A &= \frac{1}{3} \ln [k \cosh t] \pm \frac{t}{2\sqrt{6}}, \\ B &= \frac{1}{3} \ln [k \cosh t] \pm \frac{4t}{2\sqrt{6}}, \\ C &= -\frac{1}{6} \ln [k \cosh t].\end{aligned}\tag{4.8}$$

The constant k is related to the field strength at $t = 0$. As $t \rightarrow \pm\infty$, $C \rightarrow -\infty$ and thus the transverse space is like the tail of an infinite tear drop (that has the shape of Σ_6) appearing in time. The total electric flux is given by

$$\int *F = \frac{k}{2} V_6 \int_{-\infty}^{+\infty} e^{-4A+B+6C} dt = \frac{V_6}{k},\tag{4.9}$$

where V_6 is the volume of Σ_6 . Therefore, when Σ_6 is compact, the total charge is finite.

In a similar way one can construct flux SM6 brane which is given by

$$\begin{aligned}ds^2 &= e^{2A} (dx_1^2 + \dots + dx_7^2) - e^{2B} dt^2 + e^{2C} d\Sigma_3^2, \\ F_{\alpha\beta\gamma t} &= \frac{k}{2} e^{-7A} \epsilon_{\alpha\beta\gamma},\end{aligned}\tag{4.10}$$

where the indices α, β, γ refer to the tangent space of the Ricci flat manifold Σ_3 . The metric functions can be found as

$$\begin{aligned}A &= \frac{1}{6} \ln [k \cosh t] \pm \frac{t}{2\sqrt{21}}, \\ B &= \frac{1}{6} \ln [k \cosh t] \pm \frac{7t}{2\sqrt{21}}, \\ C &= -\frac{1}{3} \ln [k \cosh t],\end{aligned}\tag{4.11}$$

and the total magnetic charge is

$$\int F = \frac{k}{2} V_3 \int_{-\infty}^{+\infty} e^{-7A+B+3C} dt = \frac{V_3}{k},\tag{4.12}$$

where V_3 is the volume of Σ_3 .

As discussed in [27], there is an interplay between fluxbranes and p -branes. Namely, a fluxbrane can be described as a limit of a brane anti-brane system which is similar to the appearance of constant electric field lines in between the plates of a capacitor. One would

expect a similar relation to hold for S-branes and flux S-branes. Namely a flux S-brane should be realized as the limit of two S-branes separated by a finite time interval. It would be interesting to search for this possibility using the regular S-brane solutions constructed in [8–10].

One can work out generalizations of the above solutions where the Ricci flat space Σ is replaced by a positively or negatively curved Einstein manifold (especially a solution with a sphere looks natural for the compactness of fluxlines wrapping over it). However, as in the case of usual fluxbrane backgrounds, the differential equations cannot be decoupled due to the extra curvature terms. For instance in the flux SM3 brane solution the equation system (4.7) is modified such that

$$\begin{aligned} \ddot{A} &= \frac{k^2}{3} e^{12C} \\ \ddot{C} &= -\frac{k^2}{6} e^{12C} - 5\sigma e^{8A+10C} \\ 10\dot{A}^2 + 30\dot{C}^2 + 48\dot{A}\dot{C} - \frac{k^2}{2} e^{12C} + 30\sigma e^{8A+10C} &= 0, \end{aligned} \tag{4.13}$$

where $\sigma = +1$ and $\sigma = -1$ correspond to Σ being a unit sphere and a unit hyperboloid in (4.5), respectively. It seems impossible to decouple these equations due to σ terms which are related to curvature of Σ . In this case one can try to integrate equations numerically or search for special exact solutions. For $\sigma = -1$, we found a power law solution which can be written as

$$\begin{aligned} A &= -\frac{1}{24} \ln(\alpha t) + \beta, \\ B &= -\frac{7}{6} \ln(\alpha t) + 4\beta, \\ C &= -\frac{1}{6} \ln(\alpha t), \end{aligned} \tag{4.14}$$

where $\alpha = 2\sqrt{2k}$ and $e^{8\beta} = 3k^2/10$. Introducing the proper time coordinate $d\tau = -e^B dt$ the metric becomes

$$ds^2 = (\tilde{\alpha}\tau)^{1/2} e^{2\beta} (dx_1^2 + \dots + dx_4^2) - d\tau^2 + (\tilde{\alpha}\tau)^2 dH_6^2, \tag{4.15}$$

where $\tilde{\alpha} = 2(5/3)^{1/2}/3$. Eq. (4.15) can be viewed as the asymptotic limit of a more general solution and one can see that the integral of F converges as $\tau \rightarrow \infty$. Let us note that the transverse space parameterized by (τ, H_6) is *not* flat and there is a conic singularity as $\tau \rightarrow 0$ since $\tilde{\alpha} \neq 1$. For $\sigma = +1$, the power law ansatz does not work and we cannot find a special solution. It seems that numerical techniques should be used to integrate equations for this case.

5. Conclusions

In recent discoveries on nonperturbative aspects of string theory, p -brane solutions played a crucial role. Especially backgrounds corresponding to D-branes gave a lot of new information since their dual CFT description as open strings obeying Dirichlet boundary

conditions are known. S-brane solutions are also expected to shed some light on time dependent phenomena in string theory. In worldsheet CFT, S-branes arise when the time coordinate obeys a Dirichlet boundary condition. In terms of supergravity fields they can be described as time dependent solutions. Despite recent interesting developments it seems that to have a better understanding of S-branes more information is needed in both side of these dual descriptions.

In this paper, we construct new S-brane solutions in $D = 11$ supergravity theory. Firstly, we seek for solutions that can be characterized by a harmonic function H on the transverse space. It turns out that the Einstein's equations demand H to be a linear function. The solutions can be classified according to the codimension one hyperplane Σ_H being spacelike, timelike or null. We observe that spacelike backgrounds are identical to the previously constructed SM2 and SM5 brane solutions with flat transverse spaces. Our construction reveals two additional family corresponding to timelike and null planes. It is possible to superpose different solutions which would simply rotate or shift the plane Σ_H . The null solution preserves 16 supersymmetries of $D = 11$ supergravity and others are non-supersymmetric. We also show that the solutions can be generalized naturally with arbitrary Ricci flat Lorentzian spaces. The harmonic S-brane solutions can be thought as the generalizations of S-branes with a flat transverse space. It would be interesting to consider other cases with spherical or hyperbolic transverse spaces and find their harmonic extensions.

We should note that the solutions where Σ_H is not spacelike admit timelike or null Killing vectors and thus it is difficult to interpret them as backgrounds describing decays of unstable branes. The static solution can be thought to represent the end of the decay process since the corresponding background should be static asymptotically. On the other hand, it is natural to expect that the supersymmetric null solution is related to boosted $M2$ -branes in a certain limit. In any case, it is interesting that all these solutions can be described as different subclasses of the same family.

In supergravity brane solutions it is crucial to identify symmetries properly. For an Sp -brane in D dimensions one would expect an $SO(1, D-p-2)$ symmetry in the transverse space. However in some physical applications the symmetry groups are necessarily broken down. In terms of gauge theories living on the branes that would correspond to giving vacuum expectation values to some scalars. In supergravity description symmetries are broken in the solutions when parallel branes are separated from each other. It is not clear whether one can place two parallel S-branes separated by a finite time interval. Assuming this can be done, one can smear the time coordinate and $SO(1, D-p-2)$ symmetry should be broken down to $SO(D-p-2)$ subgroup. In this work we also construct solutions which can be thought to represent time smeared static S-branes. These backgrounds resemble black p -brane solutions in that they are asymptotically flat and support finite ADM masses. However static S-branes are not black objects since they contain generic naked singularities in the interior. They are also non-supersymmetric.

Finally, we obtain solutions for spacelike fluxbranes. A fluxbrane background has antisymmetric tensor field components tangent to the transverse space. The main characteristic property is the convergence of the total charge, although fluxlines have infinite

extent. For a spacelike fluxbrane the transverse space is Lorentzian. Not surprisingly the fluxlines now extend in time from past to future infinity but the total charge is still finite. As for timelike fluxbranes one would expect to obtain spacelike backgrounds as the limit of a solution which describes S-brane pairs separated by a finite time interval.

Various S-brane solutions have been constructed in the literature and in this paper we obtain new solutions which have interesting physical properties. We believe however that the final word on supergravity description of S-branes has not been said. Especially, one should have a clearer understanding of the relation between supergravity solutions and CFT description of S-branes. For an object that appears for a moment in time one would expect the corresponding solution to be localized both in time and in transverse spatial coordinates. In this case, however, it seems $SO(1, D - p - 2)$ symmetry should be broken. It would be interesting to study this possibility and construct purely localized S-brane solutions.

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